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COST EFFECTIVE APPROACH: OPTIMAL CONTROL APPLICATION IN NARCOTICS ABUSE MODEL BY INVOLVING THE NUMBER OF DEATHS

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Abstract

Narcotics abuse represents a critical global public health challenge requiring strategic intervention through prevention and treatment mechanisms. This study addresses a significant gap in existing mathematical models of narcotics abuse by incorporating mortality as an explicit variable. While the foundational LH (Light user–Heavy user) model has been instrumental in understanding drug abuse dynamics, it fails to account for the substantial mortality burden associated with narcotics abuse. This research develops an extended LHD (Light user–Heavy user–Death) model by adding a death state to the original LH framework. The model is calibrated using data from Indonesia's National Narcotics Agency (BNN) through parameter selection that approximates observed trends. Optimal control theory is applied to minimize prevention and treatment budgets while maximizing intervention effectiveness. Simulations were conducted under three budget scenarios: unrestricted, restricted (Rp 735 billion based on 2014 allocation), and balanced (fifty–fifty allocation). Results demonstrate that prevention strategies dominate under unrestricted budget conditions, achieving a 78.8% reduction in light users, while treatment interventions become more prominent under budget constraints, with optimal resource allocation varying significantly across scenarios. The findings provide empirical evidence for evidence-based policy formulation in narcotics control programs and contribute methodologically to the field of mathematical epidemiology by explicitly modeling mortality dynamics in substance abuse.

Keywords: Applied Mathematics, Illicit Drug; Model, Optimal Control, Cost Allocation

INTRODUCTION

Optimal control is fundamentally concerned with finding control inputs that optimize a specified objective function subject to system dynamics constraints (Yoshioka & Yaegashi, 2021). The discipline seeks to determine the best sequence of decisions over time to achieve desired goals while accounting for system constraints and performance metrics. At its core, optimal control theory provides a systematic methodology for solving dynamic optimization problems where decisions must be made sequentially over time (Song, 2024). Dynamic programming represents one of the two principal approaches to solving optimal control problems (Guerdouh et al., 2022). This methodology decomposes complex optimization problems into sequences of simpler subproblems, exploiting the principle of optimality to build solutions recursively. The dynamic programming principle yields the Hamilton-Jacobi-Bellman (HJB) equation, which characterizes the value function representing the minimum cost achievable from any given state. In 2010 Indonesia launched a Narcotics Free Indonesia in 2015, but the abuse and trafficking of narcotics became more prevalent in 2015 (Kadarudin, Thamrin, & Liao, 2018). Indonesia is not yet free from narcotics abuse. Narcotics abuse has penetrated all lines and orders of life, regardless of age, profession, or intellect (Tartila, 2022). Without realizing it, sometimes narcotics offenders live during society. It was only realized when the users were arrested by the authorities. Some of them are family, friends, or respected people in their environment. Narcotics abuse is a crime that continues to expand, seeing its abuse trend that continues to increase from year to year (Pardo & Reuter, 2018). This is indicated in the last 3 years from various facts and case arrest data, the results of interviews with key informants, and confirmations to various parties; it is even more popular and has reached rural areas (Dabney & Tewksbury, 2016). This picture shows that the prevention and countermeasures carried out are still ineffective. A mathematical model of a rug problems known as the LH model was introduced by Everingham and Rydell. In this model, there are two states, namely light abuser L (Light User) and heavy abuser H (Heavy User). This model continues to be developed and refined to answer the increasingly large and complex challenges of narcotics abuse (Volkow & Blanco, 2021). The LH model was expanded by Behren by introducing the Initiation function which states the initiation rate of the number of light users if left alone or with the existence of narcotics counseling programs by related parties either in the community or in schools. Behren further developed this model by adding a third state of E (Ever Heavy User), in other words 'Once a Heavy User' so it became known as the LHE model. This model was developed because the negative behavior of heavy narcotics users does not necessarily disappear immediately, but is affected by memory (Kutlu & Gould, 2016). The longer a person becomes a narcotics abuser, the greater the memory, but this model is not the last model in the implementation of the memory of narcotics users (Goodman & Packard, 2016). Two years before introducing the LHE model, Behren (2000) began to build an LHY model where E (Ever Heavy User) 'Ever Been a Heavy Abuser' was replaced by Y (number of Heavy User Years) 'Number of Years of Heavy Abuser'. However, Behren and Tagler (2000) state that this model is more difficult to analyze. This LHY model and the LHE model have the same difficulty in estimating parameters in states Y and E due to the absence of data. Narcotics abuse can lead to death. UNODC reported that globally in 2012 an estimated 324 million people or about seven percent of the world's population were drug users and an estimated 183,000 people died per year due to narcotics abuse worldwide.

In the LH model or other models, there is a parameter of light abuser L (Light User) which 'changes' to heavy abuser H (Heavy User) or non-abuser and heavy abuser H (Heavy User) who 'stops' (Jaureguiberry, 2018). Some of them don't just 'change' or 'stop' but die. In the LH model the element of death is not present in the model, although in fact deaths affect the number of users each year (Balk, 2004). Death of the abuser was combined in the light abuser who stopped and the heavy abuser who quit (Engel, 2023). The addition of the element of death (Death) to this LH model is very possible by adding the death parameter in the light abuser L (Light User) and the heavy abuser H (Heavy User) or the addition of the death state can also be formed so that this model is more representative of the actual situation because of the number of deaths that take thousands of human lives every year. The prevention and handling of narcotics is carried out with various actions ranging from gentle educational and persuasive actions to very harsh actions (Anugrah & Witasari, 2021). Educational and persuasive actions are carried out when counseling in educational institutions for students or students and in various places for the community, parents, and policy makers (Kaplin, Lee, Hutchens, & Rooksby, 2019). Crackdown is carried out when narcotics abuse has crossed the limits of the sustainability of life (Turkar, 2023). Prevention and handling can affect narcotics abuse, both consumption, abuse, the number of users, and the price of narcotics due to the principle of scarcity (Abd Al-Hassan, 2023). With the increasing price of narcotics, it has an impact on the less ability of people to become users so that there is less death related to narcotics abuse (Pardo & Reuter, 2018). It is undeniable that both prevention and treatment require costs. A strategy is needed to minimize the budget on handling and prevention because a country's budget is not only on narcotics issues (Patel et al., 2016). Despite the extensive literature on narcotics abuse modeling, several critical research gaps remain (Mollick & Kober, 2020). First, existing models predominantly focus on prevalence dynamics while neglecting mortality as an explicit outcome variable, thereby underestimating the true societal burden (Heesterbeek et al., 2015). Second, limited research has explored optimal resource allocation strategies in budget-constrained environments typical of developing countries (Shirzadi Javid, Omrani, & Falegari, 2025). Third, most studies utilize data from developed nations with substantially different epidemiological profiles and intervention infrastructures compared to Southeast Asian contexts (Dunn, Turner, Tun, & Anderson, 2016). Fourth, the dynamic interplay between prevention timing, treatment intensity, and mortality reduction has not been adequately characterized in existing mathematical frameworks (Heffernan, Cooke, Nayagam, Thursz, & Hallett, 2019). This study addresses these gaps by developing an LHD model that explicitly incorporates mortality, applies optimal control theory to resource allocation, and utilizes Indonesian epidemiological data to ensure contextual relevance. Based on the background of the problems described above, this study focuses on efforts to minimize the budget for the prevention and handling of narcotics to reduce narcotics abuse and reduce the number of deaths as a result. The purpose of this study is to develop a model of narcotics abuse by considering the number of deaths, complete the model to obtain optimal solutions in minimizing the cost of preventing and handling narcotics abuse, and finding optimal control strategies both in conditions of cost limitation and without cost limitations. The benefits of this research are expected to contribute to the academic world in analyzing the cost control of the prevention and handling of narcotics abuse optimally, as well as being an input for the National Narcotics Agency in designing and implementing a more efficient and effective narcotics prevention and handling program.

RESEARCH METHOD

Applied mathematics is the branch of mathematics that focuses on developing and using mathematical methods, models, and techniques to solve practical problems arising in science, engineering, business, economics, medicine, and many other fields. Unlike pure mathematics, which is primarily concerned with abstract concepts and theoretical structures, applied mathematics seeks to translate real-world phenomena into mathematical language so that they can be analyzed, predicted, and optimized.

Given the complexity of optimal control problems, particularly for nonlinear systems, numerical methods are essential for practical implementation. The value function and corresponding stochastic optimal control are often obtained numerically when the associated partial differential equation cannot be solved analytically. A sufficient condition for finding the optimal control is the verification theorem, which confirms that a candidate solution satisfies the necessary optimality conditions (Delavarkhalafi, 2022).

This study used a literature study method by studying various sources such as books, papers, journals, articles, and research results that are relevant to the prevention and handling of narcotics abuse as well as mathematical models that have been developed previously. The researcher modified the existing model by adding a new variable, namely the state of death, based on the study *Optimal Control of Drug Epidemics: Prevent and Treat–But Not At the Same Time?* by Behren et al. (2000) and the book *Optimal Control of Nonlinear Processes With Applications in Drugs, Corruption, and Terror* by Grass.

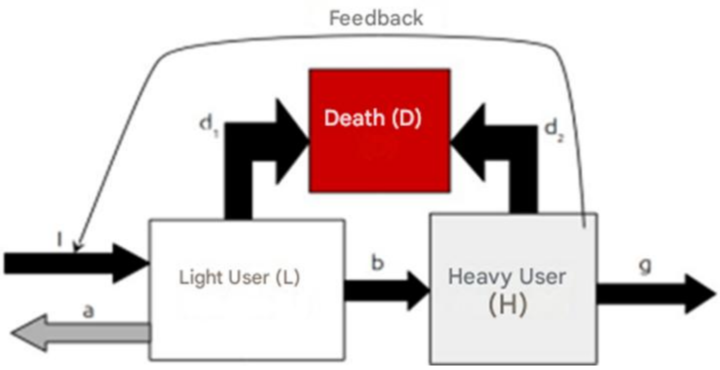

Figure 1. Model of narcotics abuse with mortality with variable L, H and D

Figure 1 The optimal model of control in narcotics with the additional *death states* above is expressed by the following equation system.

$$\frac{dL}{dt} = I(L,H) - (a + b + d_1)L$$
$$\frac{dH}{dt} = bL - (g + d_2)H$$
$$\frac{dD}{dt} = d_1L + d_2H$$

The model developed is a development of the LH model with the addition of the element of mortality (D) to describe the number of users who die per year. Control is carried out through two approaches, namely prevention (w) and handling (u) as a form of budget allocation. This model is solved numerically using the Pseudospectral and Sequential Quadratic Programming (SQP) methods with the help of TOMLAB/PROPT software under MATLAB. Furthermore, the discussion was carried out through simulations based on three scenarios from the National Narcotics Agency of Indonesia to analyze the effectiveness of the control and prevention strategy.

RESULTS AND DISCUSSION

LHD Model and Its Empirical Analysis

The LHD model is a development of the LH model. The LHD model is different from the LH model because of the variable of death (D). The LHD model places people based on narcotics abuse into four groups, namely: non-users, light users (L), heavy users (H), and users who die due to narcotics abuse (D). These four groups continue to experience changes in number every year, either in the form of addition or subtraction due to the transfer from one group to another with the rate of displacement using parameters and parameter functions as found in the following system of differential equations LHD model.

$$\frac{dL}{dt} = \psi(w)I(L,H) - (a+b+d_1)L$$
$$\frac{dH}{dt} = bL - (g+d_2)H - \beta(H,u)H$$
$$\frac{dD}{dt} = d_1L + d_2H$$
$$I(L,H) = \tau + sLe^{-q\frac{H}{L}}$$
$$\psi(w) = h + (1-h)e^{-mw}$$
$$\beta(H,u) = c\left(\frac{u}{H+\varepsilon}\right)^d$$
$$u \geqslant 0$$
$$\gamma Q - u \geqslant 0$$

with the objective of minimizing the budget for prevention and response as follows.

$$\min_{u(\cdot)}\left\{(\kappa+\gamma)\int_0^T e^{-rt}\,Qdt\right\}$$

The differential equation shows the change in light users that occurs due to 5 main factors, namely:

1. Reduced number of light users due to the presence of light users who stop and return to non-users every year, the amount of which depends on parameter *a*.
2. Decrease in the number of minor users due to an increase in the status of minor users to heavy users every year, the amount of which depends on parameter *b*.
3. Reduce the number of light users due to the presence of light users who die every year due to narcotics whose amount depends on the *d1 parameter*.
4. The number of light users increases due to the initiation of non-users to light users every year, the amount of which depends on the function of parameter *I* is stated in the following equation.
$$I(L,H) = \tau + sLe^{-q\frac{H}{L}}$$
This equation shows initiation that is influenced by two main factors, namely non-users who become light users *L* due to their own volition without invitation from anyone called Innovators *τ* and light users who attract non-users to become light users. Initiation is comparable to light users because heavy users are considered not to commit incarceration due to their circumstances that require serious treatment and are socially isolated. The annual rate of light users of interest non-users is expressed by the parameter *s*. In addition, initiation is influenced by the proportion of heavy users to light users. The greater the number of light users the greater the non-users are withdrawn and conversely the greater the heavy users the weaker initiation in the year due to the large number of heavy users who let alone to affect non-users even they need serious treatment and are busy with their high doses, in this study it is assumed that innovators *τ=10,000 people.*
5. Reduced number of light users because it is controlled by ψ prevention with a budget *w* in the initiation parameter *function I.*
$$\psi(w) = h + (1-h)e^{-mw}$$
The quation states that the assumed prevention function can only trim light users to a certain level *ψ.* In this study, the maximum prevention was only up to a level greater than 84 percent by choosing *h=0.84* as seen in Figure 1. Prevention is proportional to the amount of budget spent on prevention *and* its effectiveness *m.*

In the differential equation, it is stated that the change in the number of heavy users occurs due to 4 main factors, namely:

1. Heavy users continue to increase every year due to the increasing rate of light users whose status as a heavy abuser depends on parameter b.
2. Heavy users decrease every year due to the rate of quitting heavy users, the amount of which depends on the g parameter.
3. Heavy users decrease every year due to the rate of dead heavy users, the amount of which depends on the d2 parameter.
4. Heavy users decrease every year due to control in the form of handling β with a budget for heavy users. Treatment can reduce heavy users by β as seen in Figure 2.4 and expressed by equation (4.6) as follows.
$$\beta(H,u) = c\left(\frac{u}{H+\varepsilon}\right)^d$$

The differential equation states the change in the number of deaths of users from 2 main contributors, namely: due to heavy users and light users who die because these two types of narcotics users both experience deaths related to narcotics abuse. Thus, light users and heavy users can experience overdoses, depression, suicide, accidents and other causes that lead to death.

Budget for Simulation

Simulation of optimal control application in the narcotics abuse model by involving the number of deaths, the system is controlled or influenced by two main factors, namely prevention and handling. With prevention and handling, the number of users, abuse and the price of narcotics have changed. Prevention and handling all required costs or budgets in its implementation. In 2014 BNN received a budget allocation of Rp. 735,051,825,000,- (seven hundred and thirty-five billion fifty-one million eight hundred and twenty-five thousand rupiah) (BNN & Pulitkes UI, 2015:86). In this simulation, it will be calculated how much budget should be given or what the impact if the budget used is as large as the budget in 2014. The budget for narcotics in 2015 is not yet known for sure. Therefore, it is not used in this simulation. Every simulation with a limited budget uses the 2014 data.

Discount Rate

The application of optimal control on the narcotics abuse model by involving the number of deaths is related to the price and value of the currency that changes in value over time. The *discount rate* value used in this simulation *is r=0.075. This is in accordance with the discount rate issued* by Bank of Indonesia (BI) in 2015.

Scenario of Number of Narcotics Users

BNN & Pulitkes UI (2015:16) estimates that there are 3 (three) scenarios of the number of narcotics users in Indonesia in 2014-2020. It was simulated in the study of the application of optimal control in the narcotics abuse model by involving this number of deaths. The three scenarios are hereinafter called up, stable and downward scenarios. Estimates and projections of the number of narcotics users in the last year in Indonesia 2014-2020 (in thousands of people) in each scenario can be seen in Table 1 as follows.

Table 1. Estimation and projection of the number of narcotics users in the last year in Indonesia, 2014-2020 (in thousands of people)

Gender	Scenario	2014	2015	2016	2017	2018	2019	2020
Man	**Increasing**	3,088.7	3,224.0	3,348.7	3,461.4	3,561.5	3,648.3	3,722.8
	Stable	2,997.5	3,051.5	3,105.5	3,159.0	3,211.9	3,264.4	3,318.0
	Decreasing	2,884.6	2,837.6	2,803.8	2,783.4	2,777.4	2,786.9	2,814.0
Woman	**Increasing**	1,058.4	1,109.6	1,157.1	1,200.5	1,239.1	1,272.9	1,302.1
	Stable	1,025.2	1,046.6	1,068.1	1,089.5	1,110.4	1,131.3	1,152.5
	Decreasing	986.0	972.2	963.0	958.4	958.6	964.2	975.8

(BNN & Pulitkes UI. 2015:16)

Number of Light Abuse, Severity, and Deaths

The number of light, heavy, and fatal users is crucial in the simulation of optimal control applications in narcotics abuse models involving the number of deaths. Therefore, data on the number of light, heavy, and dead users is needed. The estimated number of narcotics users according to the level of narcotics dependence in 2014 is 4,147,100 narcotics users. A total of 1,723,975 people were including light users (try to use), the remaining 2,298,727 were heavy users. A total of 33 people die every day in Indonesia due to narcotics abuse cases. This figure was obtained from the number of 12,044 people who had died throughout 2014 due to narcotics (BNN & Pulitkes UI. 2015:55).

Number of Narcotics and Their Abuse Rate

The types of narcotics are different from one another, as well as the unit, price or dosage used, in grams, wraps, rolling, packages or milliliters. This simulation uses grams in units, therefore each type of narcotic is converted into grams. The number of narcotics in circulation in Indonesia and their conversion in 2014 is presented in Table 2 below.

Table 2. Number of estimated narcotics circulation and seizures in Indonesia 2014 (1 item = 400 mg)

Type	Sum	In grams
Cannabis (gram)	158,522,831	158,522,831
Hasish (gram)	8,873,515	8,873,515
Heroin/cell (gram)	9,284,430	9,284,430
Ecstasy (granules)	14,376,448	5,750,579
Shabu (gram)	219,837,040	219,837,040
Nipam (grain)	7,804,479	3,121,792
Koplo Pill (grain)	22,971,576	9,188,630
Rohypnol (granules)	2,999,151	1,199,660
Valium (grain)	17,960,841	7,184,336
Xanax (grain)	22,087,002	8,834,801
Cocaine (grams)	664,188	664,188
LSD (milligrams)	1,179,308	1,179
Total	-	432,462,982

(BNN & Pulitkes UI. 2015:44)

In this ccenario, the assumption is used that a heavy *abuser H* consumes 5 times the consumption of cenar, therefore the average cenary abuser consumption (grams/year) is determined as follows.

k_L=Q/(L+5*H)=432,462,982/(1,723,975+5*2,298,727)= 31.25

Meanwhile, the average consumption of heavy users (grams/year) is obtained by multiplying by 5 the average level of consumption of cenar users as below.

k_H= 5*k_L=5* 31.25=156.25

The total cost of narcotics abuse is not private costs plus social costs. Private costs 1524are costs related to the consumption and production of narcotics, while other costs related to narcotics and charged not to users but to the community are categorized as social costs. The cost for narcotics consumption is Rp 42,945,590,000,000 while the social cost that occurs is estimated to be around Rp 6,974,000,000,000 (BNN & Pulitkes UI, 2015:31). The social cost per person of narcotics consumption (rupiah per gram) can be determined as follows.

κ=Rp 6,974,000,000,000/432,462,982 =16,126.24

LHD Model Simulation

In this simulation, three scenarios of abuse by BNN will be used as shown in Table 3 below.

Table 3. Scenario of the number of narcotics users by BNN

Scenario	**2014**	**2015**	**2016**	**2017**	**2018**	**2019**	**2020**
Increasing	4,147.1	4,333.5	4,505.9	4,661.9	4,800.6	4,921.2	5,024.9
Stable	4,022.7	4,098.0	4,173.6	4,248.4	4,322.3	4,395.8	4,470.5
Decreasing	3,870.5	3,809.8	3,766.8	3,741.8	3,736.0	3,751.1	3,789.9

BNN & Pulitkes UI. 2015:16

In this simulation, the initial value is used according to data from the 2015 BNN given in Table 4 as follows.

Table 4. Initial values starting in 2014

L0	H0	D0
1,723,975	2,298,727	12,044

The simulation was carried out according to conditions in Indonesia with more heavy users than light users because users who only used (*ever used*) once in their lifetime were not included in the light users. The general mortality rate is 0.003. There is no data on the number of deaths in light users or in heavy users specifically, therefore the mortality rate in each light and heavy abuser is assumed to be the same as the general death rate of d1=0.003 and d2=0.003, respectively.

Ascending Scenario

In the upside scenario, the number of light, heavy users and deaths is estimated using the proportion of 2014. The number of light, heavy and death in the ascending scenario can be seen in Table 5 below.

Table 5. Number of light users, severity and mortality in the ascending scenario

Scenario/ Year	2014	2015	2016	2017	2018	2019	2020
Increasing	4,147.1	4,333.5	4,505.9	4,661.9	4,800.6	4,921.2	5,024.9
Light (L)	1,723.9	1,801.4	1,873.0	1,937.9	1,995.6	2,045.7	2,088.8
Heavy (H)	2,423.2	2,532.1	2,632.9	2,724.0	2,805.0	2,875.5	2,936.1
Dead (D)	12.1	12.6	13.1	13.6	14.0	14.4	14.7

The data in Table 5 above is expressed in the LHD model by selecting the values of the parameters in the model without involving the function of the control parameters of prevention and handling in such a way that it is close to the data trend in the upward scenario of the National Narcotics Agency (BNN). The data is approached in such a way that it is not below the estimated data of BNN so that there is no *underestimation* in the countermeasures of narcotics abuse as shown in Figure 2 below.

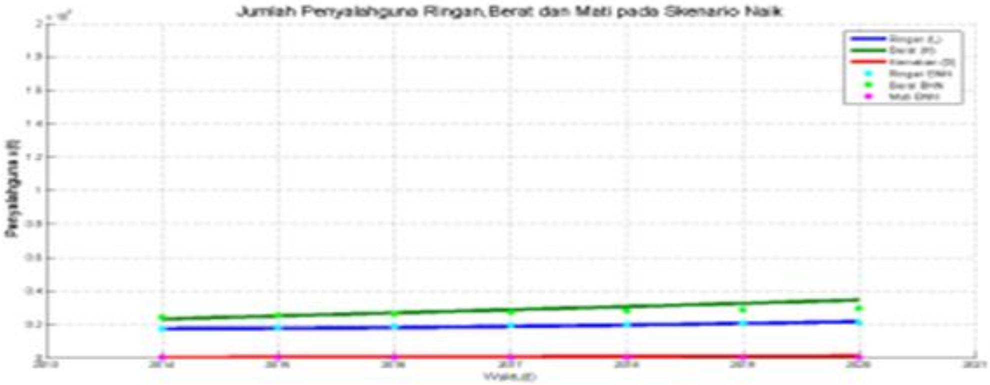

Figure 2. Light users, heavy and pre-control deaths in the ascending scenario

The parameter values that are close to the conditions of the ascending scenario are shown in Table 6 below.

Table 6. Parameter values in ascending scenarios

Parameter	Value
a	0.163
b	0.02
g	0.062
s	0.195
q	0.47
T	1x104
d1	0.003
D2	0.003

Table 6 shows the parameter values that are close to the conditions of the ascending scenario indicating some assumptions. A value of a=0.163 assumes that the annual rate of light users quit annually as much as 16.3 percent of the number of existing light users. The annual rate of light to heavy users was 2 percent of the number of light users that year. The value of g=0.062 assumes that the annual rate of heavy users quit by 6.2 percent of the total number of existing heavy users. The value of s=0.195 assumed that 19.5 percent of attractive light users were not light users becoming light users.

The number of innovators per year τ=1x104 is assumed to be 10,000 people every year of their own volition to become narcotics users without the need to be influenced by light users. The value d1=d2=0.003 shows that the number of light and heavy users who died was 0.3 percent of the entire population of heavy and light users. In this ascending scenario, a simulation will be carried out using three countermeasures budget schemes, namely: unlimited, limited and balanced budgets.

In the scenario of an unrestricted budget, the state is assumed to have an unlimited allocation of funds for the prevention and handling of narcotics, so the simulation results show a significant decrease in the number of light users from 1,723,100 to 369,400 people or a decrease of 78.8 percent, as well as a small decrease in heavy users from 2,299,000 to 2,254,400 people or 1.94 percent. The largest budget is allocated for prevention with a peak of Rp 22.54 trillion, while handling is Rp 709.4 billion. In the scenario of a budget limited to Rp 735 billion, the strategy changes with the dominance of handling over prevention; Light users decreased by 77.63 percent to 385,500 people, while heavy users decreased slightly by 1.12 percent to 2,273,200 people. The highest budget for prevention is only Rp 69.6 billion, much smaller than the handling budget of Rp 711 billion. In the balanced budget scenario (fifty-fifty), prevention and handling each get Rp 367.53 billion per year. The results showed a decrease in light users by 77.82 percent or 382,300 people, but heavy users increased by 1.65 percent to 2,337,100 people. Overall, the highest effectiveness of reducing narcotics abuse is obtained in the condition of the budget not being restricted, while the most optimal budget management efficiency is found in the balanced budget scenario.

Stable Scenario

As in the rising scenario, the number of light, heavy users and deaths in the stable scenario also used proportions in 2014. The number of light users, heavy and death in the ascending scenario can be seen in Table 7 below.

Table 7. Number of light users, severity and mortality in stable scenarios

Scenario/ Year	2014	2015	2016	2017	2018	2019	2020
Stable	4022.7	4098	4173.6	4248.4	4322.3	4395.8	4470.5
Light (L)	1723.90	1756.17	1788.57	1820.62	1852.29	1883.79	1915.80
Heavy (H)	2298.80	2341.83	2385.03	2427.78	2470.01	2512.01	2554.70
Dead (D)	12.10	12.33	12.55	12.78	13.00	13.22	13.45

The data in Table 7 above is expressed in the LHD model by selecting the values of the parameters in the model without involving the function of the preventive and handling control parameters in such a way that it is close to the data trend in the upward scenario of the National Narcotics Agency (BNN). The data is approached in such a way that it is not below the estimated data of BNN so that there is no *underestimation* in narcotics abuse as shown in Figure 3 below.

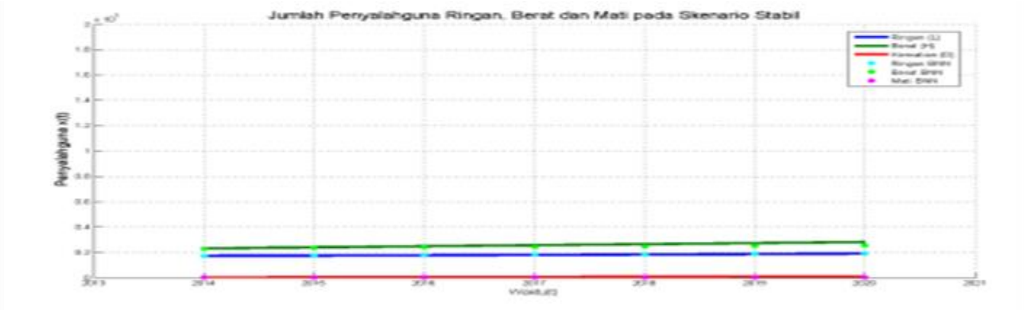

Figure 3. of light users, severity and mortality before control in stable scenarios

The parameter values that are close to the conditions of the ascending scenario are shown in Table 8 below.

Table 8. Parameter values in stable scenarios

Parameter	**Value**
a	0.163
b	0.14
g	0.062
s	0.195
q	0.34
T	1x104
d1	0.003
D2	0.003

Table 8 shows the parameter values that are close to the conditions of the ascending scenario indicating some assumptions. A value of a=0.163 assumes that the annual rate of light users quit annually as much as 16.3 percent of the number of existing light users. A value of b=0.14 assumes that the annual rate of light users becoming heavy users is 14 percent of the number of light users that year. The value of g=0.062 assumes that the annual rate of heavy users quit by 6.2 percent of the total number of existing heavy users. The value of s=0.195 assumed that 19.5 percent of attractive light users were not light users becoming light users. The number of innovators per year τ=1x104 is assumed to be 10,000 people every year of their own volition to become narcotics users without the need to be influenced by light users. The value d1=d2=*0.003* shows that the number of light and heavy users who died was 0.3 percent of the entire population of heavy and light users. In this stable scenario, a simulation will be carried out using three countermeasures budget schemes, namely: unlimited, limited and balanced budgets

The difference in parameters with the stable scenario with the ascending scenario is found in the values of parameters b and q (the annual rate of light users becoming heavy users and the constant prevention effect of heavy users), while the other parameters are the same. In a stable scenario, the value of the parameter b=0.14 is greater than in the ascending scenario b=0.02. Then the parameter value q=0.34 in the stable scenario while in the higher scenario it is greater, namely q=0.47. Because these differences cause the simulation results in the stable scenario to be different from the ascending scenario. In this stable scenario, a simulation will be carried out using three countermeasures budget schemes, namely: unlimited, limited and balanced budget. The simulation in the stable scenario is as follows.

Budget is not Restricted

In this simulation, it is assumed that the state will allocate an unlimited budget for the prevention and handling of narcotics in Indonesia, even though in real circumstances the budget is always limited. Simulations with an unrestricted budget are carried out to find out the range of budgets that should be provided to get more optimal results.

The change in the number of users resembling the results of the simulation of the budget increase scenario is not restricted. The number of light users has decreased greatly from the initial condition, the number of heavy users tends to be the same as the initial condition and the number of deaths decreases, but nevertheless the number of users is different from the simulation scenario of an unrestricted budget increase. The number of light users in the initial condition was 1,723,100 people. After five years of prevention, it will be 580,000 people, which is far behind the 369,400 people in the upward scenario. This condition has decreased by 66.34 percent. Heavy users experience a change in numbers not as large as light users. In the early period, there was a slight increase and a slight decrease until the fifth year. Initially, the number of heavy users numbered 2,299,000 while in the fifth year the number of heavy users dropped to 2,139,400 people. The change in this number decreased by 6.94 percent. The decrease in the number of heavy users to 2,139,400 people or by 6.94 percent better than the 2,254,400 people in the scenario of the budget increase is not restricted. This is affected by a larger parameter b (annual rate of light to heavy users) under stable conditions. This makes it a priority to reduce the number of heavy users greater and has an impact on reducing the number of more heavy users. The number of deaths has decreased from 12,068 people to 8,363 people. This condition decreased by 30.69 percent. The change in the number of heavy, light and fatal users is influenced by the pattern of prevention and treatment chosen as shown in Figure 4.6(ii). The budget on prevention is greater than the budget on handling. The budget for prevention experienced a very large increase in the first year and began to decrease from the second year until entering the first semester of the fifth year. This prevention time is longer than the prevention time in the scenario of an unrestricted budget increase. Treatment is carried out stably from year to year from the first year to the fifth year with a smaller budget when compared to prevention. For more clarity, Figure 4.6(iii-iv) shows the amount of budget spent on each prevention and treatment.

The amount of the budget for prevention is much larger compared to handling with details: the highest budget amount for prevention is Rp 21,801,000,000,000 (22 trillion) and the highest budget amount for handling is Rp 702,050,000,000 (709 billion). This budget, both in prevention and in handling, is both lower than in the scenario of increasing the unrestricted budget. Estimates of the number of light users, heavy users, the number of deaths, the amount of the budget for prevention and the amount of the budget for treatment are shown in Table 9 below.

Table 9. Simulation results in unrestricted budget stable scenarios

Month to	Light (S)	Heavy (H)	Death (D)	Prevention (w)	Handling (u)
0	1.0e+006 * 1.7233	1.0e+006 *	1.0e+004 * 1.2067	1.0e+013 * 0.2924	1.0e+011 *
6	1.6869	2.2988	1.1975	1.9521	7.0205
12	1.5760	2.3047	1.1686	2.1736	7.0168
18	1.6573	2.3193	1.1899	2.0890	6.9968
24	1.2236	2.3091	1.0662	2.0383	7.0127
30	1.0368	2.3304	1.0040	1.3629	6.8390
36	0.8729	2.3100	0.9437	0.6896	6.6855
42	0.7445	2.2726	0.8914	0.1590	6.4992
48	0.6549	2.2268	0.8515	0	6.3093

56	0.5901	2.1834	0.8206	0	6.1464
60	0.5800	2.1454	0.8158	0	6.0110
		2.1394			0

Budget Constraints

It is assumed that the budget allocation for narcotics eradication is the same every year. A constrained budget is more likely than an unconstrained budget. A country's budget is spent on various needs, not just on one activity. In 2014 BNN received a budget allocation of Rp 735,051,825,000 (seven hundred and thirty-five billion fifty-one million eight hundred and twenty-five thousand rupiah) (BNN, 2015:86). The budget is limited in such a way that the total expenditure on prevention and handling does not exceed the budget in 2014. The limitation is expressed in the following form:

$w+u \leq 735{,}051{,}825{,}000$ budgets on prevention and handling are subject to fluctuation within those limits.

The number of light users decreases from the initial condition, the number of heavy users tends to be the same as the initial condition and the number of deaths decreases. The number of light users in the initial condition was 1,723,100 people, after five years of prevention it became 623,900 people. This condition has decreased by 63.79 percent. Heavy users experience changes in numbers unlike light users. In the initial period, there was a slight increase and a slight decrease in the fifth year. Initially, the number of heavy users amounted to 2,299,000 while in the fifth year the number of heavy users dropped to 2,163,800 people or a decrease of 5.87 percent. The number of deaths has also decreased along with the reduction in the number of light and heavy users. Initially, the number of deaths amounted to 12,068 people contributed by light and heavy users with a mortality rate of 0.003. Every year the number of deaths continues to decrease until in the fifth year the number of deaths amounted to 8,363 people. This change in the number of heavy, light and fatal users is influenced by the pattern of prevention and treatment chosen as shown in Figure 4.7(ii). The budget for handling is very large while for prevention is relatively small. The budget for prevention has increased in the first year until entering the first semester of the fifth year. Handling is carried out in a monotonous manner from year to year from the first year to the fifth year with a dominant budget when compared to the prevention budget. For more clarity, Figure 4.7(iii)-(iv) shows the amount of budget spent on prevention and treatment, respectively.

The highest budget amount for prevention is IDR 101,990,000,000 (IDR 102 billion) and the highest budget amount for handling is IDR 702,060,000,000 (IDR 702 billion). The amount of this budget both for prevention and for handling is both lower than in the upside scenario. Estimate of the number of light users, heavy users, number of deaths, the amount of budget for prevention and the amount of budget for handling
shown in the following Table 10.

Table 10. Results of simulated light, heavy and mortality abuses in a budget-constrained stable scenario

Month to	Light (S)	Heavy (H)	Death (D)	Prevention (w)	Handling (u)
0	1.0e+006 * 1.7234	1.0e+006 * 2.2988	1.0e+004 *	1.0e+010 *	1.0e+011 *
6	1.6905	2.3047	1.2067	3.2996	7.0206
12	1.5904	2.3198	1.1986	3.3175	7.0188
18	1.6639	2.3092	1.1610	3.4495	7.0056
24	1.2648	2.3356	1.1919	3.3411	7.0164
30	1.0871	2.3195	1.0801	4.7605	6.8745
36	0.9271	2.2870	1.0220	6.1303	6.7375
42	0.7973	2.2455	0.9642	7.8437	6.5661
48	0.7032	2.2051	0.9128	9.6362	6.3869
56	0.6346	2.1692	0.8725	0	6.2298
60	0.6239	2.1638	0.8411	0	6.0979
			0.8363	0	0

The balanced budget (*Fifty-fifty*)

The balanced budget (*fifty-fifty*) is included in the Budget category limited to a budget distribution of Rp 735,051,825,000,- (seven hundred and thirty-five billion fifty-one million eight hundred and twenty-five thousand rupiah) (BNN, 2015:86) equally for prevention and handling. The balanced budget is carried out by means that the total expenditure on prevention and handling does not exceed the budget in 2014. The limitation is expressed in the following form:

$$w \leq 735{,}051{,}825{,}000/2$$

$$u \leq 735{,}051{,}825{,}000/2$$

The results of the simulation of the stable scenario are balanced with the number of light users, heavy and mortality. The number of light users has decreased sharply since the initial condition; the number of heavy users tends to be the same and decreases slightly from the initial condition and the number of deaths decreases. The number of light users in the initial condition was 1,723,100 people, after five years of prevention it became 616,700 people. This condition has decreased quite largely, namely 64.22 percent. Heavy users experience changes in numbers not as sharp as light users. In the initial period, there was a slight increase and a slight decrease in the fifth year. Initially, the number of heavy users amounted to 2,299,000 people, while in the fifth year the number of heavy users dropped to 2,223,500 people or decreased by 3.28 percent. This change in the number of heavy, light and fatal users is influenced by the prevention and treatment patterns chosen as shown in Figure 4.8(ii). The budget for handling is larger while for prevention is relatively small. The budget for prevention has increased for the first year until entering the first semester of the fifth year. Handling is carried out continuously from year to year from the first year to the fifth year with a dominant budget when compared to prevention. For more clarity, Figure 4.8(iii)-(iv) shows the amount of budget spent on prevention and treatment.

The highest budget amount for prevention is IDR 367,530,000,000 (368 billion) and the highest budget amount for handling is IDR 367,530,000,000 (368 billion). Both prevention and handling use the existing budget to the fullest. This budget, both for prevention and for handling, is both lower than for unrestricted budgets. The budget for handlers is still more dominant because of the longer duration of expenses. Estimates of the number of light users, heavy users, number of deaths, total budget for prevention and total budget for treatment are shown in Table 11 below.

Table 11. Simulation results in balanced budget stable scenarios

Moon Wed	Light (S)	Heavy (H)	Death (D)	Prevention (w)	Handling (u)
0	1.0e+00 6 *	1.0e+006 * 2.2989	1.0e+004 *	1.0e+011 *	1.0e+011 *
6	1.7234	2.3066	1.2067	3.6753	3.6753
12	1.6902	2.3272	1.1990	3.6753	3.6753
18	1.5892	2.3125	1.1635	3.6753	3.6753
24	1.6634	2.3612	1.1928	3.6753	3.6753
30	1.2608	2.3550	1.0866	3.6753	3.6753
36	1.0818	2.3313	1.0311	3.6753	3.6753
42	0.9208	2.2968	0.9756	3.6753	3.6753
48	0.7903	2.2613	0.9261	3.6753	3.6753
56	0.6960	2.2289	0.8872	0	3.6753
60	0.6273	2.2235	0.8569	0	3.6753
	0.6167		0.8521	0	0

Downward Scenario

In the downward scenario, the number of light, heavy, and fatal users is shown in Table 14 below. This data also uses the proportion of Users in 2014.

Table 12. Number of light users, severity and mortality in the drop scenario

Scenario/ Year	2014	2015	2016	2017	2018	2019	2020
Decreasing	3870.5	3809.8	3766.8	3741.8	3736	3751.1	3789.9
Light (L)	1723.90	1696.86	1677.71	1666.58	1663.99	1670.72	1688.00
Heavy (H)	2146.60	2112.94	2089.09	2075.22	2072.01	2080.38	2101.90
Dead (D)	12.10	11.91	11.78	11.70	11.68	11.73	11.85

The data in Table 12 above is expressed in the LHD model by selecting the values of the parameters in the model without involving the function of the preventive and handling control parameters in such a way that it is close to the data tendency in the upward scenario of the National Narcotics Agency (BNN).

Table 13. Parameter Values in the downstream scenario

Parameter	Value
a	0.163
b	0.12
g	0.093
s	0.195
q	0.30
T	1x104
d1	0.003
D2	0.003

Table 13 shows the values of parameters that are close to the conditions of the rising scenario, indicating some assumptions. A value of a=0.163 assumes that the annual rate of light users quit annually as much as 16.3 percent of the number of existing light users. A value of b=0.12 assumes that the annual rate of light users becoming heavy users is 12 percent of the number of light users that year. The value of g=0.93 assumes that the annual rate of heavy users is quite 9.3 percent of the total number of existing heavy users. The value of s=0.195 assumed that 19.5 percent of attractive light users were not light users becoming light users. The number of innovators per year τ=1x104 is assumed to be 10,000 people every year of their own volition to become narcotics users without the need to be influenced by light users. The value d1=d2=0.003 shows that the number of light and heavy users who died was 0.3 percent of the entire population of heavy and light users.

The difference between the parameters and the downward scenario with the upward scenario is found in the values of parameters b, g and q (the annual rate of light users becoming heavy users and the constant of the prevention effect of heavy users), while the other parameters are the same. In the downward scenario, the value of the parameter b=0.12 is greater than in the upward scenario b=0.02, then the value of the parameter q=0.34.

CONCLUSION

The study shows that adding mortality as an explicit variable in the LHD model yields a more realistic portrayal of narcotics abuse dynamics than earlier models, with optimal-control analyses demonstrating that budget constraints heavily shape intervention choices. Under an unrestricted budget, prevention-focused strategies yield the largest impact, cutting light-user numbers by 78.8%, whereas in restricted budgets typical of real settings (e.g., Indonesia), treatment interventions dominate though with a smaller overall effect on heavy users, and the balanced-budget scenario offers a feasible compromise that still achieves meaningful reductions in light users. For future research, the model would benefit from more granular mortality data by user category rather than uniform assumptions, the incorporation of dynamic or time-varying parameters to reflect evolving epidemics, the addition of new compartments such as recovered and relapsed states to illuminate long-term trajectories and rehabilitation outcomes, and the application of the framework across varied socio-economic and geographic contexts to test generalizability and tailor context-specific optimal strategies.

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